Local randomness in Hardy's correlations: Implications from information causality principle.

MD. Rajjak Gazi,[1,*] Ashutosh Rai,[2,†] Samir Kunkri,[3,‡] and Ramij Rahaman[4,§]

[1]*Physics and applied mathematics unit, Indian statistical unit, 203 B.T. Road, Kolkata-700108, India*
[2]*S.N.Bose National Center for Basic Sciences,Block JD, Sector III, Salt Lake, Kolkata-700098, India*
[3]*Mahadevananda Mahavidyalaya, Monirampore, Barrackpore, North 24 Parganas, 700120, India*
[4]*Selmer Center, Department of Informatics, University of Bergen, Bergen, P.O. Box-7803, N-5020, Norway*

Study of nonlocal correlations in term of Hardy's argument has been quite popular in quantum mechanics. Recently Hardy's argument of non-locality has been studied in the context of generalized non-signaling theory as well as theory respecting information causality. Information causality condition significantly reduces the success probability for Hardy's argument when compared to the result based on non-signaling condition. Here motivated by the fact that maximally entangled state in quantum mechanics does not exhibit Hardy's non-local correlation, we do a qualitative study of the property of local randomness of measured observable on each side reproducing Hardy's non-locality correlation, in the context of information causality condition. On applying the necessary condition for respecting the principle of information causality, we find that there are severe restrictions on the local randomness of measured observable in contrast to results obtained from no-signaling condition. Still, there are some restrictions imposed by quantum mechanics that are not obtained from information causality condition.

PACS numbers: 03.65.Nk, 03.65.Yz

I. INTRODUCTION

Violation of the Bell-type inequalities [1] by quantum mechanics show that nature is nonlocal. Nevertheless quantum correlations respect causality principle [2]. However, there are also other non-signaling post quantum correlations [3] which cannot be distinguished from quantum correlation by subjecting them to the causality principle. Though post quantum correlations are not observed in experiments, but still we do'nt understand what underlying physical principle(s) completely distinguishes quantum correlations from nonphysical post quantum correlations.

Recent studies has shown that quantum features like violation of Bell type inequalities [3], intrinsic randomness, no-cloning [4, 5], information-disturbance tradeoff [6], secure cryptography [7–9], teleportation [10], entanglement swapping [11] are also enjoyed by other post quantum no-signaling theories. On the other hand for no-signalling correlations some implausible features has also been noticed like: some no-signalling correlations would make certain distributed computational tasks trivial [12–15] and would have very limited dynamics [16]. So the study of the nonlocal correlations in the general no-signaling framework [4–17] leads us towards a deeper understanding of quantum correlations.

Very recently, non-violation of information causality (IC) [18] has been identified as one of the foundational principle of nature, it is compatible with experimentally observed quantum and classical correlations but rules out an unobserved class of nonlocal correlation as nonphysical. The principle states that communication of m classical bits causes information gain of at most m bits, this is a generalization of the no-signalling principle, the case $m = 0$ corresponds to no-signalling. Applying IC principle to non-local correlations, we get the Tsirelson's bound [19] and all correlations that goes beyond Tsirelson's bound violate the principle of information causality [18]. In [20] it was shown that though some part of quantum boundary can be derived from a necessary condition (given in [18]) for violating IC, this condition is not sufficient for distinguishing quantum correlations from all post-quantum correlations which are below the Tsirelson's bound. So it remains interesting to see if the full power of IC (some other conditions derived from IC) can eliminate remaining post-quantum correlations below the Tsirelson's bound. Along with the research in the direction of completely distinguishing the quantum correlations from rest of the nonlocal correlations, it would also be interesting to apply the known IC condition(s) for qualitative/quantitative study of certain specific features of nonlocal correlations. For instance, it was known that maximum success probability of Hardy's nonlocality argument [21, 22] under the no-signaling restriction is 0.5 [23] and within quantum mechanics the maximum takes the value 0.09 [24], then by applying the IC principle, in [25] it was shown that the upper bound on success probability reduces to 0.20717.

In the present article we apply IC condition in order to study the property of local randomness for a bipartite probability distribution which exhibits Hardy's non-locality [21, 22]. Our motivation for this study came from the fact that Hardy's non-locality argument in quantum mechanics does not work for maximally entangled state [22, 26] and at the same time for a maximally entangled state, local density matrix being completely random, both the results for a qubit are equally probable. Keeping this in mind, we asked a more general question like: for two two-level systems, how many observable and in which way, out of four entering in the Hardy's non-locality argument, can be locally random. We want to study this question in the context of probability distribution which respects an IC condition as well as in the context of quantum mechanics. We see that the applied IC condition itself imposes power-

*Electronic address: rajjakgazimath@gmail.com
†Electronic address: arai@bose.res.in
‡Electronic address: skunkri@yahoo.com
§Electronic address: ramij.rahaman@ii.uib.no

ful restriction but still it does not reproduce all the restrictions imposed by quantum mechanics. In this context, it is to be mentioned that no signalling condition does not impose any such restriction. Interestingly we observed that the applied necessary condition for respecting IC allows at most two observable, one on each side, chosen in a restricted way to be completely random, and quantum mechanics allows only one of them to be completely random.

This article is organized as follows. In Sec. II we discuss the general structure of the set of a bipartite two input-two output nonsignaling correlations. In Sec. III we restrict the the type of correlations in Sec. II by Hardy's nonlocality conditions. In Sec. IV we study the property of local randomness in Hardy's correlation, in Sec. IV A we make this study for no-signaling correlations, in Sec. IV B we study it for correlations respecting an IC condition, in Sec. IV C we work it for quantum correlations. We give our conclusions in Sec. V.

II. BIPARTITE NONSIGNALING CORRELATIONS

Let us consider a bipartite black box shared between two parties: Alice and Bob. Alice and Bob input variables x and y at their end of the box, respectively, and receive outputs a and b. For a fixed input variables there can be different outcomes with certain probabilities. The behavior of a these correlation boxes is fully described by a set of joint probabilities $P(ab|xy)$. In this article, we will focus on the case of binary inputs and outputs $(a, b, x, y \in \{0, 1\})$. Then we have a set of 16 joint probabilities defining a bipartite binary input - binary output correlation box. These types of correlations can be represented by a 4×4 correlation matrix:

$$\begin{pmatrix} P(00|00) & P(01|00) & P(10|00) & P(11|00) \\ P(00|01) & P(01|01) & P(10|01) & P(11|01) \\ P(00|10) & P(01|10) & P(10|10) & P(11|10) \\ P(00|11) & P(01|11) & P(10|11) & P(11|11) \end{pmatrix}$$

We note that since $P(ab|xy)$ are probabilities, they satisfy positivity, $P(ab|xy) \geq 0 \ \forall \ a, b, x, y$, and normalization $\sum_{a,b} P(ab|xy) = 1 \ \forall \ x, y$. Since we are to study nonsignaling boxes; i.e., we require that Alice cannot signal to Bob by her choice x and vice versa, the marginal probabilities $P_{a|x}$ and $P_{b|y}$ must be independent of y and x, respectively. The full set of nonsignaling boxes forms an eight-dimentional polytope [17] which has 24 vertices: eight extremal nonlocal boxes and 16 local deterministic boxes. The extremal nonlocal correlations have the form

$$P_{NL}^{\alpha\beta\gamma} = \begin{cases} \frac{1}{2} & \text{if} \quad a \oplus b \ = \ XY \oplus \alpha X \oplus \beta Y \oplus \gamma, \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

where $\alpha, \beta, \gamma \in \{0, 1\}$ and $\oplus$ denotes addition modulo 2.. Similarly, the local deterministic boxes are described by

$$P_L^{\alpha\beta\gamma\delta} = \begin{cases} 1 \quad \text{if} & a \ = \ \alpha X \oplus \beta, \\ & b \ = \ \gamma Y \oplus \delta; \\ 0 \quad \text{otherwise}, & \end{cases} \tag{2}$$

where $\alpha, \beta, \gamma, \delta \in \{0, 1\}$ and $\oplus$ denotes addition modulo 2. Thus we can see that any bipartite two input- two output nonsignaling correlation box can be expressed as a convex combination of the above 24 local/nonlocal vertices.

III. HARDY'S CORRELATIONS UNDER NO-SIGNALING CONDITION

A bipartite two input - two output Hardy's correlation puts simple restrictions on a certain choice of 4 out of 16 joint probabilities in the correlation matrix. One such choice is $P(11|11) > 0$, $P(11|01) = 0$, $P(11|10) = 0$, $P(00|00) = 0$ and it is easy to argue that these correlations are nonlocal. To show this, let us suppose that these correlations are local i.e. they can be simulated by noncommunicating observers with only shared randomness as a resource. Now consider the subset of those random variables λ shared between the two observers such that for λs belonging to this subset input $x = 1, y = 1$ give output $a = 1, b = 1$ (this subset is nonempty since $P(11|11) > 0$), now conditions $P(11|01) = 0$ and $P(11|10) = 0$ tell that within this subset input $x = 0, y = 0$ would give output $a = 0, b = 0$, this would imply that $P(00|00) > 0$), but it contradicts the condition $P(00|00) = 0$. Hence these correlations are nonlocal. If we further restrict these correlations by no-signaling condition we get Hardy's nonsignaling boxes. It is easy to check that these boxes can be written as a convex combination of 5 of the sixteen local vertices P_L^{0001}, P_L^{0011}, P_L^{0100}, P_L^{1100}, P_L^{1111} and 1 of the eight nonlocal vertex P_{NL}^{001}. Then,

$$\begin{aligned} P_{ab|XY}^{\mathcal{H}} \ = \ & c_1 P_L^{0001} + c_2 P_L^{0011} + c_3 P_L^{0100} \\ & + c_4 P_L^{1100} + c_5 P_L^{1111} + c_6 P_{NL}^{001} \end{aligned} \tag{3}$$

where $\sum_{j=1}^{6} c_i \ = \ 1$. From here the correlation matrix for these Hardy's nonsignaling boxes can be written as

$$\begin{pmatrix} 0 & c_1 + c_2 + \frac{c_6}{2} & c_3 + c_4 + \frac{c_6}{2} & c_5 \\ c_2 & c_1 + \frac{c_6}{2} & c_3 + c_4 + c_5 + \frac{c_6}{2} & 0 \\ c_4 & c_1 + c_2 + c_5 + \frac{c_6}{2} & c_3 + \frac{c_6}{2} & 0 \\ c_2 + c_4 + c_5 + \frac{c_6}{2} & c_1 & c_3 & \frac{c_6}{2} \end{pmatrix}$$

IV. PROPERTY OF LOCAL RANDOMNESS IN HARDY'S CORRELATIONS

For a most general bipartite correlation an input x on Alice's side is locally random if the marginal probabilities of all possible outcomes on Alice's side for this input, are equal and similarly for Bob. In the case of two input- two output bipartite correlations: an input x on Alice's side is locally random if, $P(0|x) = P(1|x) = \frac{1}{2}$, in terms of joint probabilities this would mean that for any choice of Bob's input y, $P(00|xy) + P(01|xy) = P(10|xy) + P(11|xy) = \frac{1}{2}$. Similarly an input y on Bob's side is locally random

if, $P(0|y) = P(1|y) = \frac{1}{2}$, in terms of joint probabilities this can be expressed as, for any choice of Alice's input x, $P(00|xy) + P(10|xy) = P(01|xy) + P(11|xy) = \frac{1}{2}$. Let us denote the 0 and 1 inputs on Alice's (Bob's) side as $0_A(0_B)$ and $1_A(1_B)$ respectively. We would now like to see that, what choices of inputs from the set $\{0_A, 1_A, 0_B, 1_B\}$ can be locally random for a given class of Hardy's correlations.

Input	Conditions for local randomness
0_A	$c_1 + c_2 + \frac{c_6}{2} = \frac{1}{2}$ $c_3 + c_4 + c_5 + \frac{c_6}{2} = \frac{1}{2}$
1_A	$c_1 + c_2 + c_4 + c_5 + \frac{c_6}{2} = \frac{1}{2}$ $c_3 + \frac{c_6}{2} = \frac{1}{2}$
0_B	$c_3 + c_4 + \frac{c_6}{2} = \frac{1}{2}$ $c_1 + c_2 + c_5 + \frac{c_6}{2} = \frac{1}{2}$
1_B	$c_2 + c_3 + c_2 + c_4 + c_5 + \frac{c_6}{2} = \frac{1}{2}$ $c_1 + \frac{c_6}{2} = \frac{1}{2}$

TABLE I: For the no-signaling bipartite Hardy's correlation with two dichotomic observable on either side, here each row give the conditions which coefficients c_is must satisfy for the corresponding input to be locally random.

A. Hardy's correlations respecting no-signaling

In the case of Hardy's correlations which respects no-signalling, condition of local randomness for each of the possible inputs, are given in the TABLE I. Now let us see that for the Hardy's correlations respecting no-signalling, what choices of inputs can be locally random. We give the results for every case, in the TABLE II. We can read from here that although in order to show the property of local randomness Hardy's correlations becomes much restricted, yet we get solutions for each case. If we get solutions for the case 1, it is obvious that there are solutions in all the remaining cases 2-15 , nevertheless we write the complete table giving the form of solutions in each case for the later reference.

Cases	Locally random inputs	C_1	C_2	C_3	C_4	C_5	C_6
1.	$\{0_A, 1_A, 0_B, 1_B\}$	$\frac{1}{2}(1-c_6)$	0	$\frac{1}{2}(1-c_6)$	0	0	c_6
2.	$\{0_A, 1_A, 0_B\}$	c_1	$\frac{1}{2}(1-c_6)-c_1$	$\frac{1}{2}(1-c_6)$	0	0	c_6
3.	$\{0_A, 1_A, 1_B\}$	$\frac{1}{2}(1-c_6)$	0	$\frac{1}{2}(1-c_6)$	0	0	c_6
4.	$\{0_A, 0_B, 1_B\}$	$\frac{1}{2}(1-c_6)$	0	c_3	$\frac{1}{2}(1-c_6)-c_3$	0	c_6
5.	$\{1_A, 0_B, 1_B\}$	$\frac{1}{2}(1-c_6)$	0	$\frac{1}{2}(1-c_6)$	0	0	c_6
6.	$\{0_A, 1_A\}$	c_1	$\frac{1}{2}(1-c_6)-c_1$	$\frac{1}{2}(1-c_6)$	0	0	c_6
7.	$\{0_B, 1_B\}$	$\frac{1}{2}(1-c_6)$	0	c_3	$\frac{1}{2}(1-c_6)-c_3$	0	c_6
8.	$\{1_A, 1_B\}$	$\frac{1}{2}(1-c_6)$	0	$\frac{1}{2}(1-c_6)$	0	0	c_6
9.	$\{0_A, 0_B\}$	c_1	$\frac{1}{2}(1-c_6)-c_1$	c_3	$\frac{1}{2}(1-c_6)-c_3$	0	c_6
10.	$\{0_A, 1_B\}$	$\frac{1}{2}(1-c_6)$	0	c_3	c_4	$\frac{1}{2}(1-c_6)-c_3-c_4$	c_6
11.	$\{1_A, 0_B\}$	c_1	c_2	$\frac{1}{2}(1-c_6)$	0	$\frac{1}{2}(1-c_6)-c_1-c_2$	c_6
12.	$\{0_A\}$	c_1	$\frac{1}{2}(1-c_6)-c_1$	c_3	c_4	$\frac{1}{2}(1-c_6)-c_3-c_4$	c_6
13.	$\{1_A\}$	c_1	$\frac{1}{2}(1-c_6)-c_1-c_4-c_5$	$\frac{1}{2}(1-c_6)$	c_4	c_5	c_6
14.	$\{0_B\}$	c_1	$\frac{1}{2}(1-c_6)-c_1-c_5$	c_3	$\frac{1}{2}(1-c_6)-c_3$	c_5	c_6
15.	$\{1_B\}$	$\frac{1}{2}(1-c_6)$	$\frac{1}{2}(1-c_6)-c_3-c_4-c_5$	c_3	c_4	c_5	c_6

TABLE II: For the no-signaling bipartite Hardy's correlation with two dichotomic observable on either side, here each row gives the form of solutions for the corresponding choice of inputs to be locally random.

B. Hardy's correlation respecting information causality

Let us first briefly discuss the principle of information causality (IC) [18], then we would apply it in our study of the property of local randomness for two input- two output Hardy's nonsignaling correlations. IC principle states that for two parties Alice and Bob, who are separated in space, the information gain that Bob can reach about a previously unknown to him data set of Alice, by using all his local resources and m classical bit communicated by Alice, is at most m bits.

This principle can be well formulated in terms of a generic information processing task in which Alice is provided with a N random bits $\vec{a} = (a_1, a_2, ..., a_N)$ while Bob receives a random variable $b \in \{1, 2, , ..., N\}$. Alice then sends m classical bits to Bob, who must output a single bit β with the aim of guessing the value of Alice's b-th bit a_b. Their degree of success at this task is measured by

$$I \equiv \sum_{K=1}^{N} I(a_K : \beta | b = K),$$

where $I(a_K : \beta | b = K)$ is Shannon mutual information between a_K and β. Then the principle of information causality says that physically allowed theories must have $I \leq m$. The result that both classical and quantum correlations satisfy this condition was proved in [18]. It was further shown there that, if Alice and Bob share arbitrary two input-two output nonsignaling correlations corresponding to conditional probabilities $P(ab|xy)$, then by applying a protocol by van Dam [12] and Wolf and Wullschleger [27], one can derive a necessary condition for respecting the IC principle. This necessary condition reads,

$$E_1^2 + E_2^2 \leq 1, \tag{4}$$

where $E_j = 2P_j - 1$ ($j = 1, 2$), and P_1, P_2 are defined by,

$$\begin{aligned} P_1 &= \frac{1}{2}\left[p_{(a=b|00)} + p_{(a=b|10)}\right] \\ &= \frac{1}{2}\left[p_{00|00} + p_{11|00} + p_{00|10} + p_{11|10}\right] \\ P_2 &= \frac{1}{2}\left[p_{(a=b|01)} + p_{(a\neq b|11)}\right] \\ &= \frac{1}{2}\left[p_{00|01} + p_{11|01} + p_{01|11} + p_{10|11}\right] \end{aligned} \tag{5}$$

Here it is important to note that the condition (4) is only a necessary condition (based on the protocol give in [18]) for respecting the IC principle. So a violation of (4)implies a violation of IC but the converse may not be true. In fact, it is shown in [20] that there are examples where the condition (4) is satisfied but not the IC. We now derive some one way implications about the property of local randomness for two input - two output Hardy's nonsignaling correlations. It is easy to verify that restricting Hardy's nonsignaling correlations by condition (4) and interchanging the roles of Alice and Bob we get,

$$c_6^2 + 2(c_4 + c_5)c_6 + 2(c_4 + c_5)(c_4 + c_5 - 1) \leq 0 \tag{6}$$

$$c_6^2 + 2(c_2 + c_5)c_6 + 2(c_2 + c_5)(c_2 + c_5 - 1) \leq 0 \tag{7}$$

By applying these conditions for all possible choices of inputs that can be locally random for Hardy's nonsignaling correlations (TABLE II), we get that at least one of the above two conditions are violated for the cases $1 - 8$ but for the cases $9 - 15$ we can find c_is satisfying the above two conditions. Thus for the cases $1 - 8$ we can conclude that IC is violated, hence they cannot be true in quantum mechanics also. Now we shall study the cases 9-15 in the context of quantum mechanics in the following subsection.

C. Hardy's correlation in quantum mechanics

Violation of IC for cases 1-8 implies that there are no quantum solution for these cases. To resolve the remaining cases (9-15), we consider a two qubit pure quantum state. It is to be mentioned that for two qubits, Hardy's argument runs only for pure entangled state [28]. So without loss of any generality, we consider the following two qubit state,

$$|\Psi\rangle = \cos\beta|0\rangle_A|0\rangle_B + \exp(i\gamma)\sin\beta|1\rangle_A|1\rangle_B \tag{8}$$

. Then the density matrix $\rho_{AB} = |\Psi\rangle\langle\Psi|$ can be written in terms of Pauli matrices as,

$$\begin{aligned} \rho_{AB} = \frac{1}{4}[&I^A \otimes I^B + (cos^2\beta - \sin^2\beta)I^A \otimes \sigma_z^B + (cos^2\beta - \sin^2\beta)\sigma_z^A \otimes I^B + (2\cos\beta\sin\beta)\sigma_x^A \otimes \sigma_x^B \\ &+(2\cos\beta\sin\beta)\sigma_x^A \otimes \sigma_y^B + (2\cos\beta\sin\beta)\sigma_y^A \otimes \sigma_x^B - (2\cos\beta\sin\beta)\sigma_y^A \otimes \sigma_y^B + \sigma_z^A \otimes \sigma_z^B] \end{aligned} \tag{9}$$

The reduced density matrices ρ_A and ρ_B are,

$$\rho_A = \frac{1}{2}[I + (cos^2\beta - \sin^2\beta)\sigma_z^A] \tag{10}$$

$$\rho_B = \frac{1}{2}[I + (cos^2\beta - \sin^2\beta)\sigma_z^B] \tag{11}$$

In general an observable on a single qubit can be written as $\hat{n} \cdot \sigma$ where, $\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ is any unit vector in $\mathbb{R}^3$ and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$. Then the projectors on the eignestates of these observable are,

$$P^{\pm} = \frac{1}{2}[I \pm \hat{n} \cdot \sigma] \tag{12}$$

For observable on Alice's side to be locally random,

$$Tr(\rho_A P^+) = Tr(\rho_A P^-) \tag{13}$$

similarly for observable on Bob's side to be locally random,

$$Tr(\rho_B P^+) = Tr(\rho_B P^-) \tag{14}$$

On simplifying this we find that, for a non-maximally entangled state an observable is locally random if and only if $\theta = \frac{\pi}{2}$ i.e. $\hat{n}$ is of the form $(\cos\phi, \sin\phi, 0)$. Here we would also like to mention that for a maximally entangled state any arbitrary observable shows the property of local randomness, but we know that Hardy's argument doesn't run for a maximally entangled state. This also follows from the IC principle, as for a maximally entangled state any four arbitrary observable (two on Alice's side and two on Bob's side) are locally random and we saw that if so, it violates the IC principle.

Now suppose A (0_A) and A' (1_A) are the observable on Alice's side and B (0_B) and B' $(1_{B'})$ are the observable on bob's side. Here outputs 0 and 1 will corresponds to outcomes $+1$ and -1 respectively. Then the Hardy's correlation can be written as,

$$\begin{aligned} P(A=+1, B=+1) = \cos^2\beta\cos^2\frac{\theta_A}{2}\cos^2\frac{\theta_B}{2} + \sin^2\beta\sin^2\frac{\theta_A}{2}\sin^2\frac{\theta_B}{2} \\ +2\cos\beta\sin\beta\sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\cos\frac{\theta_A}{2}cos\frac{\theta_B}{2} \\ \cos(\phi_A+\phi_B-\gamma) = 0 \end{aligned} \tag{15}$$

$$\begin{aligned} P(A=-1, B'=-1) = \cos^2\beta\sin^2\frac{\theta_A}{2}\sin^2\frac{\theta_{B'}}{2} + \sin^2\beta cos^2\frac{\theta_A}{2}cos^2\frac{\theta_{B'}}{2} \\ +2\cos\beta\sin\beta\sin\frac{\theta_A}{2}\sin\frac{\theta_{B'}}{2}\cos\frac{\theta_A}{2}\cos\frac{\theta_{B'}}{2} \\ \cos(\phi_A+\phi_{B'}-\gamma) = 0 \end{aligned} \tag{16}$$

$$\begin{aligned} P(A'=-1, B=-1) = \cos^2\beta\sin^2\frac{\theta_{A'}}{2}\sin^2\frac{\theta_B}{2} + \sin^2\beta\ \cos^2\frac{\theta_{A'}}{2}\cos^2\frac{\theta_B}{2} \\ +2\cos\beta\sin\beta\sin\frac{\theta_{A'}}{2}\sin\frac{\theta_B}{2}\cos\frac{\theta_{A'}}{2}\cos\frac{\theta_B}{2} \\ \cos(\phi_{A'}+\phi_B-\gamma) = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} P(A'=-1, B'=-1) = \cos^2\beta\sin^2\frac{\theta_{A'}}{2}\sin^2\frac{\theta_{B'}}{2} + \sin^2\beta\cos^2\frac{\theta_{A'}}{2}\cos^2\frac{\theta_{B'}}{2} \\ +2\cos\beta\sin\beta sin\frac{\theta_{A'}}{2}\sin\frac{\theta_{B'}}{2}cos\frac{\theta_{A'}}{2}\cos\frac{\theta_{B'}}{2} \\ \cos(\phi_{A'}+\phi_{B'}-\gamma) \neq 0 \end{aligned} \tag{18}$$

For these Hardy's correlation if observable A and B (0_A and 0_B) are locally random, then $\theta_A = \theta_B = \frac{\pi}{2}$, then from equation (15) we get,

$$1 + \sin 2\beta\cos(\phi_A + \phi_B - \gamma) = 0 \tag{19}$$

then this equation is satisfied only if $\sin 2\beta$ takes the value $+1$ or -1, in either case corresponding state has to be a maximally entangled state, but this cannot be a case. Therefore we conclude that observable A and B cannot be locally random in quantum mechanics. Similarly we can see that local randomness of two observable in the cases, A' and B (1_A and 0_B) and A and B' (0_A and 1_B) is also not possible.

Now we consider the case of just one observable - say $A(0_A)$ from the set $\{A, A', B, B'\}$ to be locally random (and similarly for the cases A', B, B'). Then we find that there are non-maximally entangled states and choices of observable A, A', B, B' such that one of the observable is locally random. We give an example, consider the state $\beta = \frac{\pi}{6}$, and $\gamma = \pi$, choose observable A as $\theta_A = \frac{\pi}{2}$ and $\phi_A = \pi$, A' as $\theta_{A'} = 2\tan^{-1}(\tan^2\frac{\pi}{6})$ and $\phi_{A'} = -\pi$, B as $\theta_B = \frac{2\pi}{3}$ and $\phi_B = \pi$, and B' as $\theta_{B'} = \frac{\pi}{3}$ and $\phi_{B'} = -\pi$, then it can be easily checked that for this choice of state and observable, Hardy's argument runs and the observable A is locally random. Thus by analyzing the remaining cases $(9-15)$ within quantum mechanics, we can now conclude that for a quantum

mechanical state showing Hardy's nonlocality, at most one out of the four observable can be locally random.

V. CONCLUSION

Maximally entangled state in quantum mechanics does not reproduce Hardy's correlation whereas generalized nonsignaling theory put no such restriction on the local randomness of the observable for Hardy's correlation. We study all the possibilities of local randomness in Hardy's correlation in the context of information causality condition. We observe that not only in terms of value of maximal probability of success [25], but also in term of local randomness there is gap between quantum mechanics and information causality condition. It remains to see, in future, whether some stronger necessary condition for information causality can close this gap.

Acknowledgments

It is a pleasure to thank Guruprasad Kar and Sibasish Ghosh for many stimulating discussions. AR acknowledge support from DST project SR/S2/PU-16/2007. RR acknowledge support from Norwegian Research Council.

[1] J.S. Bell, Physics **1**, 195 (1964); J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[2] G.C. Ghirardi, A. Rimini and T. Weber, Lett. Nuovo Cim. **27** (1980) 263.
[3] S. Popescu and D. Rohrlich, Found. Phys. **24**, 379 (1994); Sandu Popescu, arXiv:quant-ph/9709026 (1997).
[4] L. Masanes, A. Acin, and N. Gisin, Phys. Rev. A **73**, 012112 (2006).
[5] H. Barnum, J. Barret, M. Leifer, and A. Wilce, Phys. Rev. Lett.**99**, 240501 (2007).
[6] V. Scarani et al. Phys. Rev. A **74**, 042339 (2006).
[7] J. Barret, L. Hardy, and A. Kent, Phys. Rev. Lett. **95**, 010503 (2005).
[8] A. Acin, N. Gisin, and L. Masanes, Phys. Rev. Lett. **97**, 120405 (2006).
[9] L. Masanes, Phys. Rev. Lett. **102**, 140501 (2009).
[10] H. Barnum, J. Barret, M. Leifer, and A. Wilce, arXiv:quant-ph/0805.3553v1 (2008).
[11] P. Skrzypczyk, N. Brunner, and S. Popescu, Phys. Rev. Lett. **102**, 110402 (2009).
[12] W. van Dam, e-print arXiv:quant-ph/0501159.
[13] G. Brassard, Phys. Rev. Lett. **96**, 250401 (2006).
[14] N. Linden, S. Popescu, A.J. Short, A. Winter, Phys. Rev. Lett. **99**, 180502 (2007).
[15] N. Brunner, and P. Skrzypczyk, Phys. Rev. Lett. **102**, 160403 (2009).
[16] J. Barrett, Phys. Rev. A **75**, 032304 (2007).
[17] J. Barrett, N. Linden, S. Massar, S. Pironio, S. Popescu and D. Roberts Phys. Rev. A **71**, 022101 (2005).
[18] M. Pawlowski,T. Paterek, D. Kaszlikowski, V. Scsrani, A. Winter and M. Zukowski, Nature **461**, 1101 (2009).
[19] B.S. Tsirelson, Lett. math. Phys. **4**, 93 (1980).
[20] Jonathan Allcock, Nicolas Brunner, Marcin Pawlowaski, and Valerio Scarani, Phys. Rev. A **80**, 040103(R)(2009).
[21] L. Hardy, Phys. Rev. Lett. **68**, 2981 (1992).
[22] L. Hardy, Phys. Rev. Lett. **71**, 1665 (1993).
[23] S.K. Chaudhary, S. Ghosh, G. Kar, S. Kunkri, R. Rahaman, and A. Roy, e-print arXiv:0807.4414.
[24] S. Kunkri, S.K. Chaudhary, A. Ahanj, and P. Joag, Phys. Rev. A **73**, 022346 (2006).
[25] Ali Ahanj, Samir Kunkri, Ashutosh Rai, Ramij Rahaman, and Pramod S. Joag, Phys. Rev. A **81**, 032103 (2010).
[26] Adan Cabello, Phys. Rev. A **61**, 022119 (2000).
[27] S. Wolf and J. Wullschleger, e-print arXiv:quant-ph/0502030v1 (2005).
[28] G. Kar, Phys. Lett. A **228**, 119 (1997).